AN ABSOLUTE MEASUREMENT OF THE THOMSON EFFECT IN COPPER.

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Presented April 13, 1898, by E. H. Hall.

The experiments described in this paper were carried out at the Jefferson Physical Laboratory of Harvard University, under the supervision of Dr. E. H. Hall. A grant of one hundred dollars was obtained from the Rumford Fund of the American Academy of Arts and Sciences for the purchase of apparatus. The principle of the method employed in the experiments was devised by Prof. H. L. Callendar, M. A., F. R. S., of McGill University, and depends on the measurement of change of temperature by means of change of electrical resistance.

Historical.

In 1856 Lord Kelvin published * the details of an experiment which showed the existence of a new phenomenon, which has since been called the Thomson Effect. Aside from the interest always attached to a discovery, the Thomson effect possessed an additional interest due to the fact that, two years before, Lord Kelvin had inferred its existence by the application of the laws of thermo-dynamics to thermo-electric circuits. In spite of the great interest taken in the Thomson Effect at that time, very few attempts have been made to obtain an absolute measurement of it, or to examine its dependence on temperature conditions. This can probably be explained by its extreme smallness and difficulty of measurement.

Lord Kelvin in his original experiments sought only to prove the existence of the "electric convection of heat." The metals used in the principal experiments were iron and copper. The effect in iron was found to be considerably greater than in copper. No absolute measurements of the effect were attempted.

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* Royal Society of London, Phil. Trans., 1856.

Le Roux * in 1867 confirmed the results of Lord Kelvin. Working with a more sensitive apparatus, he obtained relative values of the Thomson Effect in several metals, and was able to show that, with a given gradient of temperature, the effect, as expected, was directly proportional to the current.

Tait, in deducing a formula to express the relation between the e. m. f. of a thermo-electric couple and the difference in temperature of the junctions, assumed that the Thomson Effect was directly proportional to the absolute temperature. The relation deduced in this way agrees very well but not exactly with the results of most experiments. In some recent experiments in which a Callendar Platinum Resistance Thermometer was used, Mr. H. M. Tory † has found a regular deviation of the experimental results from the results as given by Tait's formula.

Batelli ‡ has made absolute measurements of the Thomson Effect for a number of metals, and has found that in most cases Tait's assumption is approximately verified. As Batelli's is the only work I have found on the absolute measurement of the Thomson Effect, it is worth while to examine his method closely and to point out the possible sources of error.

Batelli prepared two rods of the metal in which the Thomson Effect was to be measured, about 30 cm. long and 0.5 cm. in diameter. One end of each rod projected into a heating bath H, the other end into a cooling bath, C, Fig. 1. There was then a flow of heat through the rods from the hot to the cold bath. The rods passed through iron boxes B_1 and B_2, 4 cm. long, containing mercury, which served as calorimeters.

When an electric current was passed up one rod and down the other, that is with and against the flow of heat in B_1 and B_2, respectively, the Thomson Effect liberated heat from one bar and absorbed it in the other. This caused the temperature of one calorimeter to rise above, that of the other to fall below, the mean temperature of the calorimeters when no electric current was flowing. From the difference in temperature of the calorimeters measured with a thermo-electric couple and the water equivalent of the calorimeters, Batelli calculated the sum of the heat liberated and absorbed in a given time by the Thomson Effect.

The method of working was as follows. The temperature of the rods was allowed to reach a steady state before the electric current was

* Annales de Chimie et de Physique. 1867.

† British Association Report. 1897.

‡ Atti della Reale Accademia delle Scienze di Torino. 1886.

applied, the mercury in the calorimeters being violently agitated meanwhile and throughout the whole experiment. (1) Readings of the difference of temperature of the calorimeters were taken at half-minute intervals for twenty minutes with no electric current flowing. (2) Readings were taken at like intervals for the same time with a known electric current flowing. (3) Readings were taken as before for twenty minutes with no current flowing.

From these three sets of observations, the actual difference in temperature between the calorimeters due to the heat liberated in one and absorbed in the other could be found. This difference in temperature between the calorimeters reached in 20 minutes, deduced from the three

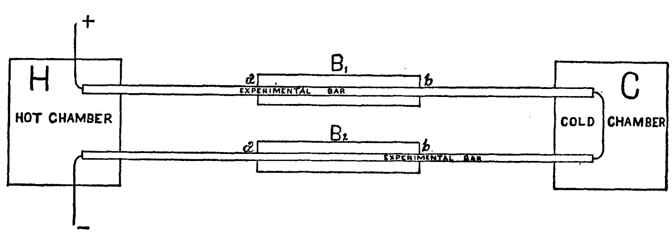

FIGURE 1.

sets of observations as described above, is multiplied by the water equivalent of the calorimeters ; the product is divided by the number of seconds in 20 minutes, by the difference in temperature between the ends of the calorimeters, that is between the points a and b, Fig. 1, by the electric current in c. g. s. measure, and by 2 to allow for the reversal of the current.

In a set of six experiments on Cadmium made as described above, the difference in temperature obtained between the calorimeters ranged between 0°.0122 C. and 0°.0085 C. The difference of temperature between the ends a, b, of the calorimeters measured in the bar was 21°.0 C.

There are two great difficulties in Batelli's method. (1) It is necessary to measure accurately a mean difference of 0°.012 C. between boxes of mercury, each of which surrounds a part of a bar in which there exists a difference of temperature of 21°.0 C. It seems likely that in spite of violent stirring considerable differences of temperature would be found in the mercury. (2) The presence of the violently agitated mercury in the calorimeter tends to destroy the temperature gradient in the rod, on which the Thomson Effect depends. In fact, it seems that the best

result in the measurement of the difference of temperature between the calorimeters would be attained only when the mercury was sufficiently agitated to make the temperature of the part of the bar surrounded by the calorimeter uniform and the Thomson Effect consequently zero. It also follows that the value obtained for the Thomson Effect would depend on the rate of stirring of the mercury. The variation between the several values found for the difference in temperature between the calorimeters sometimes, as in the specimen quoted above, amounts to 50%. It would seem from this that the difficulties mentioned above are real.

Two points of less importance are: (1) No correction was made for the change in the heat liberated by the Joule Effect due to the change in temperature, and hence in resistance of the part of the rods in the calorimeters. (2) No correction was made for the change in the rate of heat conducted into or out of the calorimeters, due to the slight change in the temperature gradient caused by the action of the Thomson Effect.

General Description of Method of this Paper.

The ends of the copper wire, in which the Thomson Effect was to be measured, were fastened into copper blocks provided with a water circulation to keep them at a constant temperature. The copper wire was heated by an electric current. There was then a flow of heat from the middle of the wire to each end. In one half of the wire this flow of heat was opposite in direction to the flow of electricity, in the other half it was in the same direction. Consequently in one half of the wire the Thomson Effect liberated heat and in the other half absorbed it. On reversing the electric current the effect was of course reversed. By measuring the change of resistance of short sections of the wire on reversal of the current, it was possible to determine the change of temperature. By a separate experiment, with the same apparatus, the relation between the heat lost by radiation and convection from any part of the wire and the temperature of that part was determined. With these data, considering any part of the wire made up of a whole number of sections, the following quantities could be determined. (1) The change in the heat generated by the Joule Effect on reversal of the electric current. (2) The change in the heat lost by radiation and convection on reversal. (3) The change in the heat conducted out of the cold end of the part of the wire under consideration on reversal. (4) The change in the heat conducted in at the hot end on reversal. These quantities suffice for the absolute measurement of the Thomson Effect.

General Description of the Apparatus.

The copper wire $a\,b$ is soldered into holes in the copper blocks B_1, B_2, Fig. 2. A brass tube, T, is slipped over the blocks and the rubber rings r_1, r_2, r_3, r_4. This tube has its sides cut away opposite the wire $a\,b$; but the ends are left whole, so that, by means of them and of the rubber rings, jackets, through which a water circulation can pass, are formed around the copper blocks. The leads, 3, 4, 5, etc. are of no. 40 B. & S. copper wire and are fastened to the bar $a\,b$ by an electrolytic deposit of copper. These fine leads are led through holes in the part of the brass tube T not cut away, and through a hard rubber strip, R, to copper binding posts on the base board. The small tube t can be

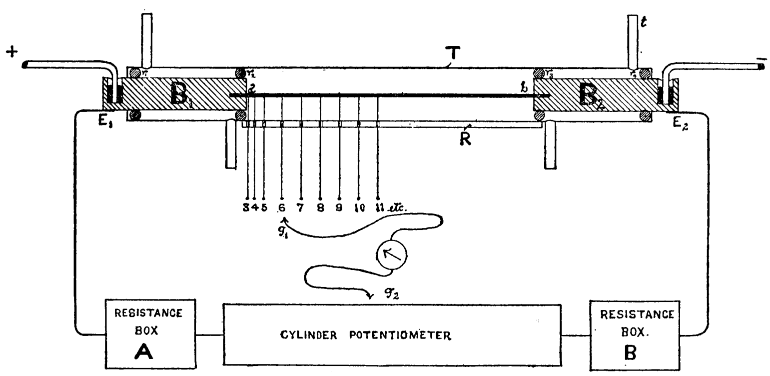

FIGURE 2.

unscrewed and a water jacket, slotted longitudinally to allow the rubber strip R and the fine leads to pass, slipped over the tube T. So that during an experiment the surroundings of the experimental wire can be kept at a constant temperature. The electric current for heating the wire is brought in by the mercury cups in the ends of the blocks B_1, B_2. Copper leads are clamped on these blocks at E_1 and E_2 and are connected to the resistance boxes A and B and cylinder potentiometer as shown in Fig. 2. One terminal, g_2, of a sensitive low resistance galvanometer is connected to the movable contact of the potentiometer; the other, g_1, can be connected with any one of the leads 3, 4, 5, 6, etc.

Working of the Method.

Suppose the wire $a\,b$ is heated by the electric current and that the blocks B_1 and B_2 and the surroundings have reached a steady temper-

ature. By adjusting the resistance in A and B the potentiometer can be made to correspond to the resistance of any section of the experimental wire. For instance, suppose the galvanometer lead g_1 is connected with lead no. 9, and that under these circumstances a balance of the galvanometer is obtained with g_2 reading nearly 100 on the potentiometer; then suppose that a balance for no. 8 is obtained at nearly 0 on the potentiometer. The difference between the potentiometer readings, that is, nearly the whole length of the potentiometer, will then correspond to the resistance of the section 8 to 9. Repeating the operation with the heating current in the opposite direction, a somewhat different length on the potentiometer will be found to correspond to the resistance of the section 8 to 9. The difference between these two lengths corresponds to the change of resistance of section 8 to 9 on the reversal of the current, on the same scale that the mean of the two lengths corresponds to its absolute mean resistance. Hence the change of temperature of any section of the wire, on reversal of the current, can be calculated. Also if the mean absolute resistance be determined, from a separate experiment, the absolute change of resistance and hence the difference in the rate of generation of heat, by the mere overcoming of resistance, in section 8 to 9, on reversal of the current, can be calculated. If from another experiment the relation between h, the heat dissipated by radiation and convection from any part of the surface of the wire, and the temperature of that part be known, then the difference in the rate of dissipation of heat by the section on reversal of the current can be calculated. There remains, for the complete determination of the Thomson Effect in section 8 to 9, to be found the change in the temperature gradients at the points 8 and 9, due to the small change of temperature on the reversal of the current.

Let us suppose that by the method described above the change of temperature on reversal of the current has been found for every section of the bar, i. e. 3 to 4, 4 to 5, 5 to 6, etc. If the results are plotted a curve such as $c\,d$, Fig. 3, is obtained. Let the curve $a\,e$ represent the distribution of temperature along the wire. The curve $c\,d$ and $a\,e$ are drawn to different scales. When the largest ordinate of $a\,e$ was 235° that of $c\,d$ was about 1°.2. The curve $c\,d$ being a temperature difference curve for corresponding points on $a\,e$, a little reflection will show that the tangent to $c\,d$ at any point, measured in degrees per cm., will be the change in the tangent to $a\,e$ at the corresponding point, on the reversal of the current. Hence by multiplying this difference tangent by the area of cross section of the bar and by the thermal con-

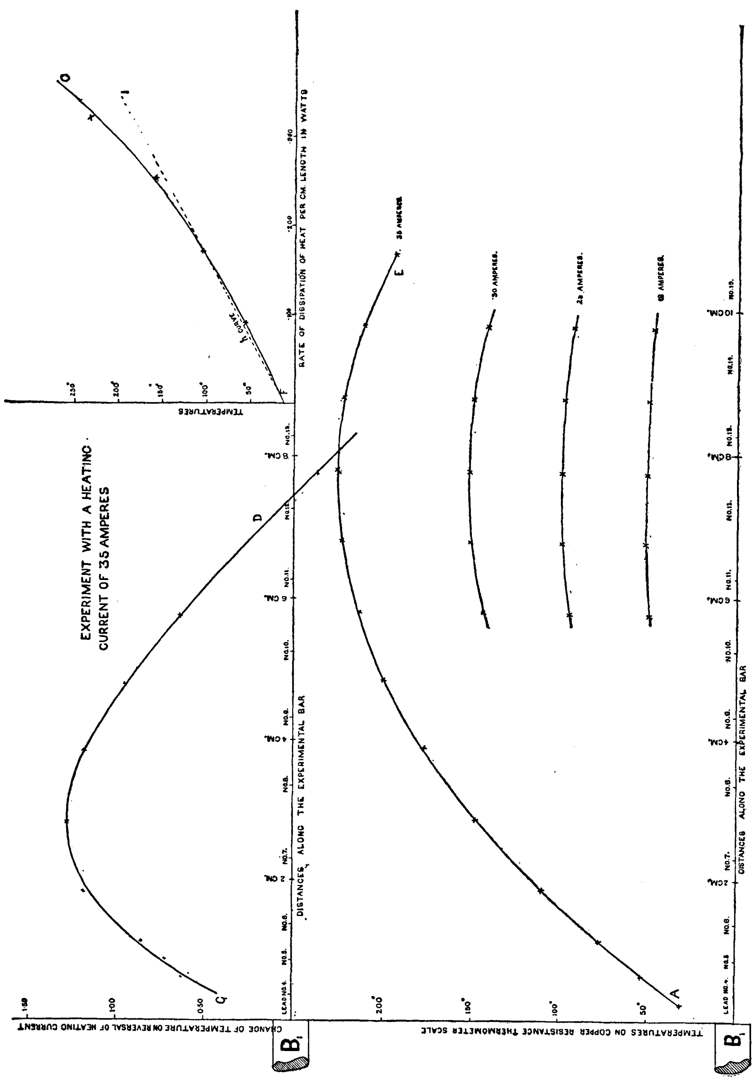
EXPERIMENT WITH A HEATING
CURRENT OF 35 AMPERES
RATE OF DISSIPATION OF HEAT PER CM. LENGTH IN WATTS
TEMPERATURES
DISTANCES ALONG THE EXPERIMENTAL BAR
CHANGE OF TEMPERATURE ON REVERSAL OF HEATING CURRENT
TEMPERATURES ON COPPER RESISTANCE THERMOMETER SCALE
35 AMPERES
30 AMPERES
25 AMPERES
O
I
D
E
A
Q
F
B₁
B₁

FIGURE 3.

ductivity at the point of the bar under consideration, it is possible to calculate the change in the amount of heat conducted past any point on reversal of the heating current. In this way the change in the amount of heat conducted out at No. 8 and No. 9 would be determined.

In practice it is advisable to use more than one short section in determining the Thomson Effect. In the experiment as set up the sections were made about 1 cm. in length, with the exception of some half as long near the ends. A part of the wire comprising at least four sections was used in calculating the value of the Thomson Effect.

For convenience of reference, let the quantities used in calculating the Thomson Effect be designated as follows: —

Let x_1 and x_2 be any two points on the experimental wire, whose temperatures are θx_1 and θx_2.

Let σ be the Thomson Effect coefficient defined by the relation $\sigma(\theta x_1 - \theta x_2)$ = heat developed in x_1 to x_2 when one c. g. s. unit of electricity passes from x_1 to x_2.

Let $R x_1 x_2$ = resistance of section (x_1 to x_2), current entering B_2.

Let $H x_1 x_2$ = heat dissipated from (x_1 to x_2), current entering B_2.

Let $k x_1 \left(\frac{d\theta}{dx}\right)$ = heat conducted past x_1 out from section, current entering B_2.

Let $k x_2 \left(\frac{d\theta}{dx}\right)$ = heat conducted past x_2 into section, current entering B_2.

Let the same letters with dashes represent similar quantities when the current enters B_1.

Let Υ = the electric current in amperes.

Then: —

$$2\sigma(\theta x_2 - \theta x_1)\frac{\Upsilon}{10} = (H^1 x_1 x_2 - H x_1 x_2) - \Upsilon^2 (R^1 x_1 x_2 - R x_1 x_2) + k x_1 \left[\left(\frac{d\theta}{dx}\right)^1 - \left(\frac{d\theta}{dx}\right)\right] - k x_2 \left[\left(\frac{d\theta}{dx}\right)^1 - \left(\frac{d\theta}{dx}\right)\right].$$

The Curve of Distribution of Temperature.

This curve has been referred to as *a e*, Fig. 3, and is obtained from a separate experiment with the same apparatus, with the exception that a standard low resistance is required in addition. The determination of this curve involved the measurement of the resistance of the short sections of the experimental wire, (1) when a current small enough not to heat the wire appreciably was flowing, (2) when the heating current Υ was flowing. From the change of resistance due to the heating current the

change of temperature was calculated. The standard low resistance was included, with the experimental wire, between the terminals of the potentiometer, as shown in Fig. 4.

The galvanometer was balanced, see Fig. 4, with the terminal g_1 connected with 1, 2, 3, 4, 5, etc., successively, by adjusting the position of the contact g_2 on the potentiometer. The relation of the differences between the potentiometer readings, corresponding to 3, 4, 5, 6, etc., to

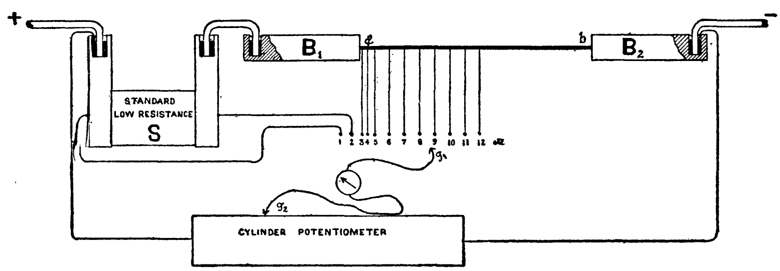

FIGURE 4.

the difference between the readings for 1 and 2, expresses the relation of the resistances of sections 3 to 4, 4 to 5, 5 to 6, etc., to the resistance of the standard S.

Determination of the h Curve, f g (Fig. 3).

If the experimental wire *a b* be long enough, the curve of distribution of temperature will be flat for the part corresponding to the middle of the bar. That is, the middle section will not be losing heat by conduction to the neighboring parts of the bar. Therefore the heat generated by the Joule Effect in the middle section will be all got rid of by radiation and convection. By measuring the resistance of the middle section and the electric current in amperes, under the above conditions, the heat generated, as well as the temperature, for the section can be calculated. By using several values for the electric current, different temperatures are obtained for the middle of the bar and corresponding values for the heat dissipated by radiation and convection. This gives all the data required for the determination of the *h* curve, provided we assume that the condition of the surface of the bar is uniform and that all parts of the bar are affected in the same way by convection currents.

In the experiments described in this paper the experimental bar was too short to avoid the effect of metallic conduction on the middle section,

so that a correction had to be made for the heat thus carried off. A knowledge of the temperature gradients at the ends of the section and the thermal conductivity was required for the application of this correction. Different observers have obtained very different values for the thermal conductivity of copper; therefore, in order to see what effect an error in the estimation of this quantity would have on the final value for the Thomson Effect, the part of the work depending on this has been worked through with two different values.

Precautions against Thermo-electric Effects at the Junctions of Dissimilar Metals.

The average resistance of a section of the experimental bar was about 0.00022 ohm. In the measurement of so small a resistance, and particularly the change in this resistance on reversal of the heating current, it was extremely important to avoid errors due to thermo-electric disturbances at junctions of dissimilar metals. The potentiometer was specially designed for the experiment and was made of 100 turns of manganine wire wound on a marble cylinder. All connections to the manganine were of copper and the instrument was completely enclosed in a glass covered box. All adjustments of the contact point could be made without opening the box. The resistance boxes A and B, Fig. 2, were also enclosed in glass covered boxes.

The fine potential leads, 3, 4, 5, etc., Fig. 2, were copper and were copper-plated to the experimental bar. The connection with the potentiometer was made through copper binding posts. The galvanometer key and circuit were entirely of copper.

It was found that copper and manganine form a thermo-electric couple only about one eighth as strong as copper and German silver, hence the choice of manganine for the potentiometer wire.

The Heating Current.

In the experiment on the change of resistance on reversal of the current, it was very important that the current used for heating the wire should be kept very nearly constant, since a change in it would cause a change in the temperature of the bar and hence in the resistance of any section. If this change were confounded with the change due to the Thomson Effect, serious error would result. The following arrangement for measuring and regulating the current was found to be very satisfactory.

A mercury trough resistance, a rough tin plate resistance and a Kelvin current balance were included in the circuit of the heating current. The mercury trough resistance was capable of being varied continuously by means of amalgamated sliding copper contacts. The slider on the Kelvin balance being set to read the desired current, the pointer on the balance arm was kept exactly at the middle mark throughout the experiment, by adjusting the sliding contacts of the mercury trough resistance. The current was drawn from ten one-hundred-ampere hour accumulators in parallel, specially set up for the experiment.

The Standard Low Resistance.

This was specially made for the experiment and consisted of four sheets of manganine in parallel between heavy copper terminals. The resistance of the standard was .000733 ohm, so that with the largest current used, that is, 35 amperes, heat was generated at the rate of about one watt. For the dissipation of this energy there was a surface of about 275 square cm. immersed in oil through which there was a good water circulation.

The Experimental Bar and Potential Leads.

The bar was of best quality copper magnet wire. Chemical analysis showed a percentage of 99.3 copper.

The length between the blocks B_1 and B_2 was 15.6 cm. The diameter was somewhat irregular, as shown by the following : —

	End.	Half-way.	Middle.	Half-way.	End.
Vertical diam.	.0952	.0960	.0956	.0962	.0966 cm.
Horizontal "	.0954	.0966	.0960	.0966	.0970 "

The bar was annealed and its surface blackened by dipping, while hot, in an acid solution of silver nitrate.

Nineteen potential leads of no. 40 B. & S. copper wire were copper plated to the bar. Their positions were as below : —

No. of Lead.	Distance from B.	Name of Section.	Length of Section.	No. of Lead.	Distance from B.	Name of Section.	Length of Section.
3	0.00 cm.	....	....	12	7.26 cm.	11 to 12	.99 cm.
$3\frac{1}{2}$	0.06 "	3 to $3\frac{1}{2}$	.06 cm.	13	8.27 "	12 to 13	1.01 "
4	0.37 "	$3\frac{1}{2}$ to 4	.31 "	14	9.28 "	13 to 14	1.01 "
5	0.86 "	4 to 5	.49 "	15	10.28 "	14 to 15	1.01 "
6	1.38 "	5 to 6	.52 "	16	11.30 "	15 to 16	1.02 "
7	2.31 "	6 to 7	.93 "	17	12.30 "	16 to 17	1.00 "
8	3.35 "	7 to 8	1.04 "	18	13.28 "	17 to 18	.98 "
9	4.33 "	8 to 9	.98 "	19	14.30 "	18 to 19	1.02 "
10	5.25 "	9 to 10	.92 "	20	14.80 "	19 to 20	.50 "
11	6.27 "	10 to 11	1.02 "	21	15.28 "	20 to 21	.48 "

Calculation of the Temperature.

In estimating changes of temperature by means of changes of resistance, using Callendar's method,* it is necessary to know the average value of the temperature coefficient for the copper between 0° and 100°, or the ratio $\frac{R_1}{R_0}$.

Where

$$R_1 = \text{resistance at } 100° \text{ C.},$$
$$R_0 = \text{resistance at } 0° \text{ C.}$$

Let $R =$ the observed resistance.
Let $Cu =$ temperature on copper resistance scale.
Let $t =$ temperature on air thermometer scale.

The ratio $\frac{R_1}{R_0}$ was taken $= 1.422$.

Temperatures are calculated according to the formula

$$Cu = \frac{R - R_0}{R_0} \times \frac{100}{\frac{R_1}{R_0} - 1} \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

* H. L. Callendar, Phil. Trans., 1887.

The relation between temperatures on the air thermometer scale and temperatures by the copper resistance thermometer is expressed by the formula

$$t - Cu = \delta \left\{ \left(\frac{t}{100}\right)^2 - \frac{t}{100} \right\} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The value of δ was taken $= 1.32$. The values of $\frac{R_1}{R_0}$ and δ given here were found in some experiments on copper magnet wire made two years ago. A small inaccuracy in the value of these constants is not important for the work described in this paper; so it was not considered necessary to determine them for the wire actually used.

The copper resistance thermometer and air thermometer scales agree, of course, at 0° and 100°; but at all other temperatures within the range of work described here there is a slight divergence, perhaps enough to make it worth while to take it into account.

Experiment with a Heating Current of 35 Amperes.

Distribution of Temperature along the Bar.

This was determined as described on page 360 the resistance of the sections of the bar being measured in terms of the standard S, Fig. 4. (1) With the heating current of 35 amperes. (2) With a current of 2 amperes. It was assumed that the current of 2 amperes did not raise the temperature of the bar above that of its surroundings. It will be seen that this assumption is allowable when we consider that 35 amperes heated the bar to a temperature of about 220°, and that the heating is proportional to the square of the current.

Referring to page 369, column V. gives the observed resistances of the sections of more than half the bar, with a current of 2 amperes, and consequently at the temperature of the surrounding water jackets, that is 11°.9 C. = 12°.1 *Cu*. Column VI. gives the observed resistances with the heating current of 35 amperes. Column IV. gives R_0, the calculated value of the resistance of each section at 0°. This is found from the resistance at 12°.1 *Cu*, the ratio $\frac{R_{12.1}}{R_0}$ being taken $= \frac{1 + 12.1 \times .00422}{1}$. Column IX. gives the temperature of each section calculated by the formula

$$Cu = \frac{R - R_0}{R_0} \times \frac{100}{\frac{R_1}{R_0} - 1}.$$

The temperatures found thus are plotted, *a e*, Fig. 3, and give a very smooth curve.

The Change of Resistance of the Sections on Reversal.

This change was measured for all the sections from lead no. 4 to lead no. 14, according to the general method as described on page 358. In order to show the exact way of taking the observations, the readings for one section are given in full below. The other sections are given only in abstract below.

Take section (9 to 10). With 72 ohms in resistance box A and 200 ohms in box B, potential lead no. 9 balanced near the zero end of the potentiometer and no. 10 near the 100 end. The readings are marked, to show the order in which they were taken, by the small numbers in parentheses.

	Lead No. 9.	Direction of Current.	Lead No. 10.	Direction of Current.
	5.412 (1)	S	91.200 (2)	S
	4.447 (3)	N	90.044 (4)	N
	5.418 (5)	S	91.208 (6)	S
	5.453 (7)	N	90.050 (8)	N
	5.422 (9)	S	91.204 etc.	S
	4.451	N	90.050	N
	5.412	S	91.204	S
	4.451	N	90.046	N
	5.422	S	91.202	S
Means :				
	5.417	S	91.204	S
	4.450	N	90.050	N
Means, plus calibration correction of potentiometer :				
	5.455	S	91.241	S
	4.488	N	90.089	N

Hence a length on the potentiometer of $91.241 - 5.455 = 85.786$ represents the resistance of section 9 to 10 when the current is in the S. direction, and a length $90.089 - 4.488 = 85.601$ represents the resistance when the current is in the N. direction. So that the difference between these lengths, that is $85.786 - 85.601 = 0.185$, represents the change of resistance on reversal of the current on the same scale that the mean of the lengths, i. e. 85.69, represents the actual resistance of the section. From column VI., page 369, the mean value of the resistance

of section 9 to 10 when carrying 35 amperes is seen to be 0.0003781, so that the absolute change in resistance on reversal is

$$\frac{.185}{85.69} \times .0003781 = .000000817 \text{ ohm.}$$

From column IV, the resistance of the section at 0° is seen to be 0.00020304, hence the change of temperature corresponding to the above change of resistance

$$= \frac{.000000817}{.00020304 \times .00422} = 0^\circ.955 \ Cu.$$

The several quantities discussed above, that is, the potentiometer turns representing the whole resistance of the several sections, the turns representing the change of resistance, the actual change of resistance in ohms, the corresponding change of temperature in degrees, are tabulated in columns II., III., VII., VIII., respectively.

The numbers representing the change of temperature on reversal, for the several sections when plotted, give the curve *c d*, Fig. 3.

The h Curve.

Observations were taken on the five middle sections of the bar, using heating currents of 35, 30, 25, and 18 amperes, in order to obtain short lengths of the temperature distribution curve at several temperatures. Unfortunately the specimen was not long enough to avoid metallic conduction of heat from the middle section, so that all the heat generated in this section is not lost by radiation and convection. The slight temperature gradient at the ends of the middle section, 12 to 13, is shown in Fig. 3.

Length of section 12 to 13 = 1.01 cm.
Mean area of cross section = .00720 sq. cm.
The thermal conductivity = .870 (1 + .0004 t).*

With the above data and the temperature gradients at the ends of the section, measured graphically, the amount of heat conducted away from the section for the several values of the heating current is obtained.

The following table gives the data required for the plotting of the h curve: —

* From some unpublished experiments.

Temperature of the Section.	Resistance of the Section.	Current in Amperes.	Rate of Heat generated, Watts.	Same per cm.	Rate of Heat conducted, Watts.	Rate of Heat dissipated per cm. length.
233.4	.0004480	35.0	.538	.533	.215	.318
157.9	.0003763	30.0	.339	.336	.084	.252
105.0	.0003259	25.0	.204	.202	.033	.169
56.9	.0002800	18.0	.091	.090	.000	.090

Plotting the values found for the rate of dissipation of heat as abscissas and the corresponding values of the temperature as ordinates, the curve $f\,g$, Fig. 3, is obtained.

Determination of the Change in the Rate of Dissipation of Heat by Radiation and Convection on Reversal of the Heating Current.

From the h curve, described in the last paragraph, and the curve of distribution of temperature along the bar, the condition of any section of the bar as regards temperature and rate of dissipation of heat can be obtained. If now the temperature of a section be slightly changed, the corresponding change in the rate of dissipation of heat can be determined by simple proportion, since the h curve is nearly a straight line. The change of temperature on reversal of the heating current has been already determined for all the sections of the part of the bar under consideration; so it is a very simple matter to calculate the change in the rate of dissipation of heat, on reversal of the current, for any section of the bar.

Column IX., page 369, gives the mean temperature of every section of the part of the bar under consideration. Column X. gives the corresponding rate of dissipation of heat from the surface of the bar, as obtained from the h curve. The change in the rate of dissipation for the several sections is tabulated in column XI. From this column the quantity $H^1 — H$, see page 369, can be at once obtained for any part of the bar consisting of a whole number of sections.

I. Section.	II. Potentiometer Turns representing Resistance of Section.	III. Turns representing Change of Resistance.	IV. Ro.	V. Resistance at 2 Amperes.	VI. Resistance at 35 Amperes.	VII. Change of Resistance on Reversal.	VIII. Change of Temperature on Reversal.	IX. Mean Temperature.	X. Mean Rate of Dissipation of Heat.	XI. Change in Rate of Dissipation of Heat on Reversal.
4 to 5	62.15	.135	.00010412	.00010943	.00012780	.000000277	0.630	53.9	.041	.00048
5 to 6	77.63	.211	.00011723	.00012321	.00015590	.000000424	0.857	78.2	.066	.00072
6 to 7	79.55	.269	.00020430	.00021471	.00030078	.000001018	1.182	111.9	.169	.00179
7 to 8	87.34	.290	.00022460	.00023615	.00036774	.000001219	1.283	151.0	.246	.00209
8 to 9	85.10	.242	.00021180	.00022195	.00037204	.000001058	1.186	180.5	.267	.00175
9 to 10	85.69	.185	.00020304	.00021340	.00037809	.000000817	0.955	204.3	.274	.00128
10 to 11	95.42	.135	.00022005	.00023128	.00042354	.000000599	0.645	219.1	.318	.00094
11 to 12	95.90	.091	.00022369	.00023510	.00044062	.000000401	0.425	229.8	.325	.00060
12 to 13	89.87	.026	.00022572	.00023724	.00044801	.000000130	0.136	233.4	....	....
13 to 14	85.50	.101	.00021797	.00022909	.00042912	.000000506	—.550	230.1	....	....
14 to 15	....	....	.00021746	.00022854	.00041863	....	....	219.2	....	....

Evaluation of the Thomson Effect.

I. Consider the sections from lead no. 4 to lead no. 12. 4 and 12 may replace x_1 and x_2 (see page 360).

Then :—

(1) $\underset{4\text{ to }12}{R^1} - \underset{4\text{ to }12}{R} = .000005813$ (from column VII., page 369). $\Upsilon = 35$;

hence $\Upsilon^2 \left(\underset{4\text{ to }12}{R^1} - \underset{4\text{ to }12}{R}\right) = 35^2 \times .000005813 = .00713$ watts.

(2) $\underset{4\text{ to }12}{H^1} - \underset{4\text{ to }12}{H} = .00965$ (from column XI., page 369).

(3) The change in the temperature gradient at no. 4 on reversal of the heating current, determined from curve $c\,d$, Fig. 3, $= 0^\circ.752$ per cm.

Area of cross section at no. $4 = .00713$ sq. cm.

The thermal conductivity at no. $4 = .89$, on the assumption that thermal conductivity $= .87\ (1 + .0004\ t)$.

Hence $k_4 \left[\left(\frac{d\,\theta}{d\,x}\right)^1 - \left(\frac{d\,\theta}{d\,x}\right)\right] = .752 \times .00713 \times .89 \times 4.2 = .0200$ watts

Similarly $k_{12} \left[\left(\frac{d\,\theta}{d\,x}\right) - \left(\frac{d\,\theta}{d\,x}\right)^1\right] = -.400 \times .00724 \times .95 \times 4.2 = -.0116$ watts.

The temperature at no. $4 = 41^\circ$.

The temperature at no. $12 = 232^\circ$.

Hence the difference of temperature between the ends of the section is 191°. Substituting in the equation of page 360, we have

$$2\ \sigma\ (232-41) \times 3.5 = .00965 - .00713 + .0200 - (-.0116)$$

$$\therefore \sigma = \frac{.00965 - .00713 + .0200 + .0116}{2 \times 3.5 \times 191} = .00002553 \text{ watts.}$$

$$= .00000608 \text{ calories per second.}$$

Mean temperature of section $= 116^\circ$ C.

Expressing the result exactly:—When an electric current of 10 amperes flows, in the same direction as the flow of heat, for 1 second through a section of a copper bar whose ends differ by 1° C. and whose mean temperature is 116° C., the Thomson Effect will be represented by the liberation of .00000608 calories.

The work of calculating the value of the Thomson Effect for other parts of the bar at different temperatures is precisely similar to the above; so a summary only is given in the following table:—

Section.	$k_{x_1}\left[\left(\frac{d\theta}{dx}\right)^1-\left(\frac{d\theta}{dx}\right)\right]$	$k_{x_2}\left[\left(\frac{d\theta}{dx}\right)-\left(\frac{d\theta}{dx}\right)^1\right]$	H^1-H
4 to 12	0.0200	− 0.01160	0.00965
6 to 12	0.01132	− 0.01160	0.00845
7 to 12	0.00356	− 0.01160	0.00666
8 to 12	− 0.00291	− 0.01160	0.00457

Section.	$Y^2(R^1-R)$	$\theta_{x_2}-\theta_{x_1}$	Mean Temperature.	σ
4 to 12	0.00713	191°	166°	.00000608
6 to 12	0.00626	141°	183°	.00000606
7 to 12	0.00500	101°	197°	.00000566
8 to 12	0.00352	65°	208°	.00000508

The values of the Thomson Effect deduced above decrease with an increase of temperature. This is, of course, contrary to the accepted view. It is, then, important to determine to what extent the necessary errors of the method used affect the result.

It is probably apparent to the reader that the most serious errors are likely to be found in the estimation of the thermal conductivity and in the determination of the h curve. As already explained, on account of the shortness of the experimental bar a large correction has to be applied for the conduction of heat from the middle section of the bar in the determination of the h curve. However, this correction is not so large that a considerable error in the estimation of the thermal conductivity would affect the value of the h curve within the limits of accuracy of this work.

The form of the h curve, fg, Fig. 3, was unexpected. It was thought that it would be either straight or bent downwards at the higher temperatures. In some experiments made two years ago at McGill University, a number of "h curves" were obtained for bars under similar conditions and in all cases they were very nearly straight. Taking, then, the dotted line fi, Fig. 3, to be the h curve, and taking its inclination from the lower points, the portion of the Thomson Effect determination affected by this change has been reworked. The results are given in column II. of the following table.

Any error in the value taken for the thermal conductivity is directly felt in the determination of the change in the rate of heat conducted in and out of the ends of the sections, on reversal of the heating current. For this reason, the value of the thermal conductivity of copper determined recently by R. W. Stewart,* namely 1.12 (1 — .001 t), has been applied to the Thomson Effect calculations. This value of the thermal conductivity differs very much from that previously used, the temperature coefficient also, being of the opposite sign. The values of the Thomson Effect as modified by this change are given in column III. below.

Mean Temperature of Sections.	I. Original Values, from page 371.	II. Values, assuming h curve to be a straight line.	III. Values, as in II., but also taking thermal conductivity = 1.12 (1 — .001 t).
166°	.00000608	.00000623	.00000678
183°	.00000606	.00000630	.00000637
197°	.00000566	.00000600	.00000564
208°	.00000508	.00000551	.00000493

It seems to be unlikely that the decrease in the Thomson Effect with rise of temperature can be accounted for by even large variations in the form of the h curve or in the value of the thermal conductivity.

A good criterion of the general accuracy of the whole scheme of the experiment is the calculation of the thermal conductivity from the several quantities involved.

Consider section 4 to 12 :—The temperature gradient, $\frac{d\,Cu}{d\,x}$, at no. 4, measured on the curve, $a\,e$, Fig. 3, is 53°.6 $Cu.$ per cm. This must be reduced to degrees Centigrade per cm.

From equation (2), page 365, differentiating we have

$$\frac{d\,t}{d\,Cu} = \frac{1}{1 - .0132\,(.02\,t - 1)}.$$

At no. 4, $t = 41$; hence $\frac{d\,t}{d\,Cu} = \frac{1}{1.024}$, therefore $\frac{dt}{d\,x} = \frac{53.6}{1.024} = 52.3$.

Let $K =$ the thermal conductivity at no. 4, i. e. at 41°. Then section 4 to 12 is losing heat by conduction at no. 4 at the rate of

* Phil. Trans. Roy. Soc. Lond. 1895.

$K \times 52.3 \times .00714 \times 4.2$ watts, .00714 being the area of cross section in sq. cm., and 4.2 expressing the relation between calories and watts.

The resistance of section 4 to 12 is .0025660 ohms, see page 369, column VI. Hence heat is being generated at the rate of $35^2 \times .0025660 = 3.145$ watts.

The rate at which heat is dissipated from section 4 to 12, by radiation and convection = 1.706 watts (see page 369, column X.), if $f\,g$, Fig. 3, be taken as the h curve.

On the assumption that the h curve is straight, heat would be dissipated at the rate of 1.951 watts.

A small quantity of heat flows into the section at no. 12. It amounts to .1070 watts.

Collecting the quantities, we have .

$$K \times 4.2 \times .00714 \times 52.3 = 3.145 + .1070 - 1.706.$$

$$\therefore K = \frac{1.546}{4.2 \times .00714 \times 52.3} = .98 \text{ c.g.s. units.}$$

Taking $f\,i$, Fig. 3, as the h curve, we should have $K = .83$ c.g.s. units.

The closeness of these values to the generally accepted ones is a good test of the general accuracy of the work, especially when it is considered that the experiment was not designed with a view to determining the thermal conductivity.

Experiment with a Heating Current of 30 Amperes.

This experiment was made on the end of the bar which had not been used in the experiment with 35 amperes, in order to see what effect a considerable change in the important quantities involved in the work would have on the results.

The data required for the calculation of the value of the Thomson Effect are tabulated on page 375.

The distribution of temperature along the bar was determined in the same way as before and a very smooth curve obtained, see $a\,e$, Fig. 5. The temperatures, column X., page 375, are calculated from the resistances given in columns IV. and VI.

The change of temperature of the sections on reversal of the heating current is considerably smaller than for the experiment with 35 amperes and the curve cd, Fig. 5, is not as good as the corresponding one in Fig. 3. The curve cd is plotted from the figures in column VIII. The quantities in column IX., from which the change in the rate of heat dissipated is obtained, are taken from the smooth curve.

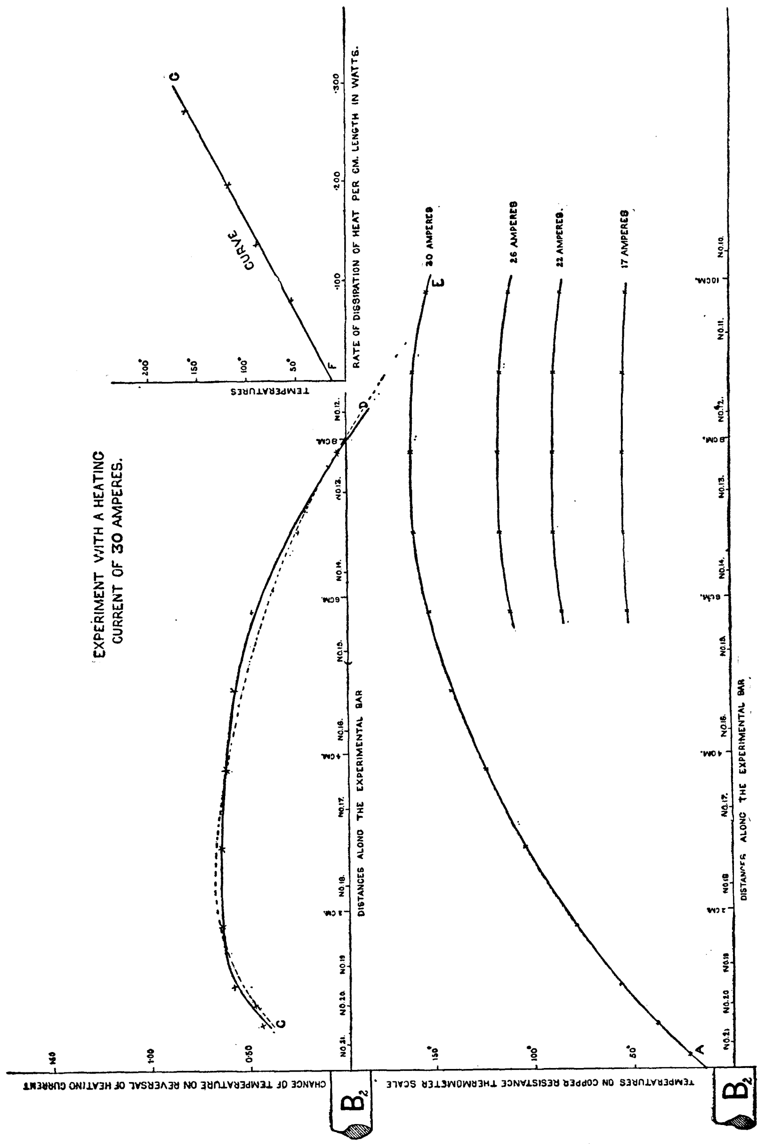
EXPERIMENT WITH A HEATING CURRENT OF 30 AMPERES.
CURVE
RATE OF DISSIPATION OF HEAT PER CM. LENGTH IN WATTS.
TEMPERATURES
DISTANCES ALONG THE EXPERIMENTAL BAR
CHANGE OF TEMPERATURE ON REVERSAL OF HEATING CURRENT
TEMPERATURES ON COPPER RESISTANCE THERMOMETER SCALE
30 AMPERES
26 AMPERES
22 AMPERES
17 AMPERES

FIGURE 5.

I. Section.	II. Potentiometer Turns representing Resistance.	III. Turns representing Change of Resistance.	IV. Calculated R_0.	V. Observed Resistance at 2 Amperes.	VI. Observed Resistance at 30 Amperes.	VII. Change of Resistance on Reversal.	VIII. Corresponding Change of Temperature on Reversal.	IX. Same by Smooth Curve.	X. Temperature.	XI. Rate of Dissipation of Heat.	XII. Change in Rate of Dissipation of Heat.
20 to 21	67.750	.108	.00010535	.00011162	.00012233	.000000195	0.439	0.413	38.2	.0216	.000233
19 to 20	51.160	.102	.00010751	.00011391	.00013333	.000000266	0.586	0.570	56.9	.0395	.000396
18 to 19	85.720	.174	.00021382	.00022654	.00028538	.000000579	0.642	0.642	79.3	.1224	.000991
17 to 18	81.090	.153	.00020832	.00022072	.00030031	.000000567	0.645	0.645	104.6	.1646	.001014
16 to 17	83.570	.141	.00021109	.00022365	.00032230	.000000543	0.610	0.616	124.8	.2050	.001012
15 to 16	81.690	.123	.00022977	.00024345	.00036718	.000000552	0.570	0.568	141.7	.2407	.000967
14 to 15	79.620	.099	.00021775	.00023071	.00035799	.000000445	0.485	0.454	152.6	.257	.000766
13 to 14	80.490	.047	.00021771	.00023066	.00036494	.000000213	0.232	0.272	160.2	.274	.000465
12 to 13	82.860	.008	.00022606	.00023951	.00038011	.000000037	0.039	....	....	....	....

The method of calculating the Thomson Effect from the above data has already been fully explained; so the principal quantities only are given in the table below:—

Section.	$k_{x_1}\left[\left(\frac{d\theta}{dx}\right)^1-\left(\frac{d\theta}{dx}\right)\right]$	$k_{13}\left[\left(\frac{d\theta}{dx}\right)-\left(\frac{d\theta}{dx}\right)^1\right]$	H^1-H	$\theta_{13}-\theta_{x_1}$	Mean Temperature of Section.	σ
20 to 13	.00696	−.00850	.005611	115.1	122	.00000628
19 to 13	.00302	−.00850	.005215	98.2	128	.00000509
18 to 13	.00000	−.00850	.004224	69.7	138	.00000483
17 to 13	−.00056	−.00850	.003210	47.1	146	.00000676

The h curve was determined by observations on the same section as in the experiments with 35 amperes. Referring to Fig. 5, fg is the curve obtained; it is quite straight and agrees with the h curve of Fig. 3 very well, as far as it goes. The bend in the latter depends principally on the observation at 35 amperes, which was, of course, not taken in this experiment.

The results of this experiment are of the same general character as in the experiment with 35 amperes. There is as before a decrease in the value of the Thomson Effect for higher temperatures, with the exception of the shortest section, that is, 17 to 13. This is due to the very flat top on the curve $c\ d$, Fig. 5. The curve $c\ d$ in this experiment is not nearly so good as in the experiment with 35 amperes. This is unfortunate, as the whole calculation for the Thomson Effect with 30 amperes depends on this curve.

For this reason, another determination of the change of temperature, on reversal of the heating current, was made for the several sections, again using 30 amperes. More observations than before were taken, so that it is hoped the means are more accurate. Great care in every respect was taken. The resulting curve is shown dotted in Fig. 5. Its general form is more similar to the curve $c\ d$ for 35 amperes, than the previous one, although the values of its ordinates do not differ greatly from it, the curves crossing each other at several points.

The principal quantities required for the calculation of the Thomson Effect as determined from this new curve are given below.

Section.	$k_{x_1}\left[\left(\frac{d\theta}{dx}\right)^1 - \left(\frac{d\theta}{dx}\right)\right]$	$k_{13}\left[\left(\frac{d\theta}{dx}\right) - \left(\frac{d\theta}{dx}\right)^1\right]$	$H^1 - H$
20 to 13	.00611	−.00840	.00548
19 to 13	.00506	−.00840	.00511
18 to 13	.00045	−.00840	.00410
17 to 13	−.00164	−.00840	.00305

Section.	$\theta_{13} - \theta_{x_1}$	Mean Temperature of Section.	σ
20 to 13	115°.1	122°	.00000597
19 to 13	98°.2	128°	.00000557
18 to 13	69°.7	138°	.00000496
17 to 13	47°.1	146°	.00000511

These results for the Thomson Effect seem to be considerably better than the first ones found with a heating current of 30 amperes. The value obtained for the shortest section and highest temperature is still a little larger than the one immediately preceding it; but it is a good deal smaller than the value found for the lowest temperature.

In order to see what effect a different value of the thermal conductivity would have on the character of the results as regards variation with temperature, the observations for the first experiment with 30 amperes have been worked through, using Stewart's value of the thermal conductivity. The modified results are given in column II. below. The original results, for the sake of comparison, of the first and second experiments are given in columns I. and III. respectively.

I. Values of σ from first experiments with 30 Amperes.	II. Values from I, modified by a change in value of Thermal Conductivity.	III. Values of σ from second experiments with 30 Amperes.	Mean Temperature of Section.
.00000628	.00000689	.00000597	122°
.00000509	.00000530	.00000557	128°
.00000483	.00000483	.00000496	138°
.00000676	.00000674	.00000511	146°

Comparison of the Experiments at 30 and 35 Amperes.

It is seen from the above results that the change made in the thermal conductivity has had the effect of increasing the diminution of the value of the Thomson Effect with a rise in temperature. The results of the two experiments with 30 amperes are not so consistent and regular as could be wished, yet it must be remembered that the reduction of the heating current from 35 to 30 amperes has probably lessened the accuracy of the work 50%. Taking from the experiment with 35 amperes the value of the Thomson Effect obtained with the largest number of sections, we have $\sigma = .00000608$ at 166° C. From the first experiment with 30 amperes we have $\sigma = .00000628$ at 122° C. These values agree fairly well, but exemplify the diminution of the Thomson Effect with a rise in temperature. From two experiments conducted on different parts of the experimental bar and with all the important quantities, such as temperature gradients etc., so different, it

is satisfactory to obtain practically the same numerical value of the Thomson Effect and with one exception the same decided evidence of the diminution of the Thomson Effect with a rise of temperature.

Accuracy of the Resistance Measurements.

On page 371 it was pointed out that the decrease in the value of the Thomson Effect with rise of temperature could not be explained by even large errors in the assumed value of the thermal conductivity or in the form of the "*h* curve." Omitting these two sources of error, the balance of the work depends on the accuracy of the resistance measurements. The accurate measurement of such low resistances as 1 cm. lengths of no. 18 copper wire, presents considerable difficulty. However, an examination of the values obtained for the same sections of the bar at different times and under different temperature conditions will show that some success was attained.

Section.	April 24th, 1897.		May 5th, 1897.		Difference.
	I. Observed Resistance at 11°.9 C.	II. Resistance at 0° calculated from I.	I. Observed Resistance at 13°.9 C.	II. Resistance at 0° calculated from I.	
10 to 11	.00023128	.00022005	.00023304	.00021995	.00000010
11 to 12	.00023510	.00022369	.00023703	.00022372	.00000003
12 to 13	.00023724	.00022572	.00023951	.00022606	.00000034
13 to 14	.00022909	.00021797	.00023066	.00021771	.00000026
14 to 15	.00022854	.00021746	.00023071	.00021775	.00000029

These figures show a very satisfactory accuracy, the agreement in one case being as close as 1 part in 7,000, the worst agreement being 1 part in 700.

Comparison of the Results with those of Batelli.

Owing to the fact that Batelli made no experiments with copper, no direct comparison with his results can be made. He obtained for iron a value of .00001215 at a temperature of 108° and his values increased very rapidly with the temperature. This result is roughly twice as great as the value found above for copper. From the slope

of the lines on the thermo-electric diagram given in J. J. Thomson's recent book on electricity and magnetism, it is seen that the Thomson Effect in iron should be about 5 times as great as in copper. Of course the determination of relative values in this way depends on making Tait's assumption that the Thomson Effect is directly proportional to the absolute temperature. If this assumption be true the determination of the Thomson Effect for different metals is very simple after its absolute value for one has been found. In the experiments described above, copper was chosen to be this particular one, because its properties suit the experimental conditions. As the results of the experiments are not in accord with Tait's assumption, it would not be consistent to deduce values of the Thomson Effect for other metals in the usual way. The further investigation of these very fundamental points is very desirable.

In closing, I wish to thank the authorities of the Jefferson Physical Laboratory for their kindness, and especially Dr. Hall for his advice and personal help with many difficult points.